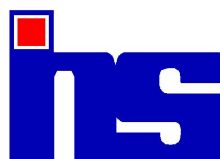


Interpolations for standard life expectancies by single year for computation of DALYs.

Dr. Prasanta Mahapatra

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Calculation of the years lost due to premature mortality (YLL) component of the disability adjusted life year (DALY) measure requires standard expectation of life at respective ages of death. Technical basis of the disability adjusted life year measure has been described by Murray (1994). The formula for YLL is

$$YLL = - \left[\frac{C e^{-\beta a}}{(\beta + r)} [e^{-(\beta + r)L}] (1 + (\beta + r)(L + a)) - (1 + (\beta + r)a) \right]$$

where L = Standard expectation of life at age a , and all other notations are same as in Murray's original article. Since disability weight (D) for premature mortality is one it has been dropped from the DALY formula to give the YLL formula above.

In the above formula the age at death a is determined by the local mortality experience. The ages at death within five year intervals, will thus vary from region to region. Even if some uniform age at death were to be used, these would be the mid points of five year age intervals. The Coal and Guo (1989) standard life tables are published by five year age groups. To read off standard life expectancies at mid point of various age intervals a complete life table by single year age group is required. Since the age at death number can have fractions of a year a complete life table by fractions of a year at least upto one decimal place would be ideal. In practice, however, we first compute a complete life table by single year age groups. Standard life expectancy at an age of death involving fraction of a year is then obtained by linear interpolation using the life expectancy at the integer part of the age at death and the next age interval. In this note I describe construction of complete life tables from the Coal and Guo abridged life tables using different interpolation methods and compare their results. Using a least squares criterion of fit, I choose one method of interpolation.

Murray (1994) has chosen the Coal and Guo model west level 26 for females as the standard for females. This has a life expectancy at birth equal to 82.5 years. For males he chooses a life expectancy of 80 years. Since none of the Coal and Guo model tables for males have exactly 80 years of life expectancy at birth he uses the female table at level 25 as the standard for males. So the interpolation exercise below is limited to these two tables.

METHODS OF INTERPOLATION

The following three methods are explored:

1. Piece wise exponential survivorship functions within five year age intervals,
2. Six point Lagrangian interpolation for ages up to 0-74 and Gompertz curve for older ages,
3. Mortpak UNABR, which uses the eight point Helligman and Pollard formula.

The method for six point Lagrangian interpolation for ages 0-74 and Gompertz curve there after has been described by Elandt-Johnson and Johnson (1980, p111-114). The Mortpak UNABR procedure is described in the Mortpak manual (United Nations, 1988) and Helligman and Pollard (1980). I used the Mortpak UNABR procedure to generate results by this method. This procedure accepts q_x data only upto 80-85 year age interval. Discussions about the exponential survival distribution can be found in any text on survival analysis (for example Elandt-Johnson and Johnson, 1980). Essentially it is assumed that the hazard rate (instantaneous probability of death) remains constant over time. This is unrealistic for the total human mortality experience. Since the abridged life table already captures most of the age pattern of mortality, modelling a constant hazard rate within the five year age groups is usually a reasonable approximation. Further details of implementation of this method in this instant case is given below. In addition to the above three methods the interpolated life expectancy by single years used by Murray for the global burden of disease (GBD) estimates was obtained. The one for males was available. So in addition to the above three methods the figures used by Murray is also included in the comparison.

The exponential survivor function within an age interval say from l_x to l_{x+n} of the abridged life table is given by $l_{x+n} = l_x e^{-t\lambda_x}$ where t = time in years after x and λ_x is the constant hazard rate within the interval (i.e. the "piece"). We estimate λ_x using the l_x and l_{x+n} numbers from the abridged life table, using the above formula which can be rearranged as follows:

$$\lambda_x = -\frac{1}{n} \ln\left(\frac{l_{x+n}}{l_x}\right).$$

Once λ_x is obtained, the l_{x+t} values can be generated for each year within the interval by using the formula $l_{x+n} = l_x e^{-t\lambda_x}$ for $t = 1, 2, \dots, 99$ and $x = 1, 5, 10, \dots, 95$. In this manner the l_x column is constructed for single year intervals using the l_x values from the abridged table.

For construction of complete life table the fraction of the last age interval lived by those dying within that interval a_x is required in addition to the single year l_x values as obtained above. The published Coal and Guo abridged tables do not provide a_x values. However these can be derived from other columns of the table using the following relationship

$$a_x = \frac{L_x - nl_{x+n}}{nd_x} = \frac{L_x - nl_{x+n}}{n(l_x - l_{x+n})}.$$

The a_x values thus obtained were used for each year within the five year age intervals.

The above procedure resulted in a life expectancy at birth of 79.89 against the original of 89 for level 25 and 82.41 against the original of 82.5 years for level 26. There is a discrepancy of 0.11 and 0.09 years respectively. The discrepancy is due to unavailability of a_x values by single year. To maintain exact correspondence with the standard life table figure the difference was added back at all ages of the complete life table.

Degree of fit of the different interpolations were measured by computing the squared deviation of interpolated values from the original values at the overlapping age groups (i.e. the age groups like 0, 1, 5, 10, ..., 100 for which both original and interpolated numbers are available). The one with the least sum of squared deviation is judged to have the best fit.

RESULTS

Comparative statement of life expectancies at different ages in the original Coal and Guo table and different interpolation results are given in annexure-1 tables a and b for level 25 and 26 respectively. The MortPak UNABR over estimates life expectancies. For example UNABR interpolated life expectancy at birth is 80.76 against original of 80 years and 84.21 against the original of 82.5 years for level 25 and 26 respectively. Similar differences persist at other ages. On the other hand the six point Lagrangian method underestimates life expectancy at corresponding ages. This method gives a life expectancy at birth of 77.81 years for the original of 80 years for level 25 and 79.12 years compared to the original of 82.5 for level 26. Table-1 here shows the fitting criterion from different interpolations. The piece wise

exponential method provides the best fit. If the discrepancy of 0.11 and 0.09 years were not added back, the sum of squared deviations for this method would be 0.31 and 0.22 for level 25 and 26 respectively. This would still leave the piece wise exponential method with the best fit to the original. Lack of fit of the six point Lagrangian and Mortpak method was anticipated. Both these methods assume Gompertzian mortality pattern at older ages. Gompertz pattern assumes that mortality rate monotonically increases for ages 75 years and beyond. Coal and Guo recognized that this is not borne out by actual experience in very low mortality populations. Consequently they modified the projection method used in the Caol and Demeney regional model life tables. For the latest low mortality models Coal and Guo incorporated a linearly decreasing hazard rate starting at 80 years. This is based on observed pattern from low mortality populations. In other words hazard rate at older ages increase up to 80 years of age and then decreases monotonically till the cohort exhausts itself completely by 110 years (Coal and Guo, 1989).

The interpolated figures currently being used (i.e. Murray's) by the Harvard Burden of Disease Unit (BDU) has fairly good fit. But the accuracy of DALY computations with respect to the adopted standard can be improved further by using the interpolated figures now arrived at by the piece wise exponential method. Thus replicability of the burden of disease estimation method can be achieved by adopting the interpolated standard life expectancy by single year now arrived at. For convenience of future work, the interpolated standard life expectancy numbers are furnished in annexure-2. These values are used in the BDAP software as the default standard life table.

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ANNEXURE-1

COMPARISON OF DEGREE OF FIT

Table-a Comparison of the degree of fit of each interpolation for Coal and Guo West L25F (COALGUOL25F)					
Age	Original	Piece wise exponential	Six point Lagrangian	Mortpak	Murray
0	80	80	77.81	80.76	80
1	79.48	79.48	77.28	80.24	79.36
5	75.62	75.62	73.42	76.38	75.38
10	70.69	70.68	68.48	71.45	70.4
15	65.74	65.74	63.53	66.5	65.41
20	60.83	60.83	58.62	61.59	60.44
25	55.94	55.94	53.72	56.69	55.47
30	51.06	51.05	48.84	51.82	50.51
35	46.19	46.19	43.96	46.97	45.56
40	41.37	41.36	39.13	42.14	40.64
45	36.61	36.61	34.36	37.37	35.77
50	31.95	31.94	29.67	32.68	30.99
55	27.4	27.4	25.09	28.12	26.32
60	22.98	22.97	20.61	23.74	21.81
65	18.73	18.73	16.28	19.61	17.5
70	14.72	14.72	12.11	15.79	13.58
75	11.1	11.1	8.46	12.37	10.17
80	8.04	8.03	6.29	9.41	7.45
85	5.67	5.64	4.54	6.94	5.24
90	3.93	3.88	3.18	4.99	3.54
95	2.73	2.68	2.21	0	2.31
100	1.93	2.04	1.93	0	1.46
Sum of squared deviations:		0.02	95.09	27.34	10.8

Table-b Comparison of the degree of fit of each interpolation for Coal and Guo West L26F				
Age	Original	Piece wise exponential	Six point Lagrangian	Mortpak
0	82.5	82.5	79.12	84.21
1	81.84	81.83	78.44	83.55
5	77.95	77.95	74.55	79.67
10	72.99	72.98	69.58	74.7
15	68.02	68.01	64.61	69.73
20	63.08	63.08	59.67	64.79
25	58.17	58.17	54.76	59.88
30	53.27	53.27	49.85	54.99
35	48.38	48.38	44.96	50.11
40	43.53	43.52	40.09	45.25
45	38.72	38.72	35.27	40.44
50	33.99	33.99	30.51	35.69
55	29.37	29.36	25.84	31.04
60	24.83	24.83	21.24	26.55
65	20.44	20.43	16.75	22.27
70	16.2	16.2	12.35	18.26
75	12.28	12.28	8.46	14.6
80	8.9	8.89	6.3	11.35
85	6.22	6.2	4.55	8.57
90	4.25	4.21	3.19	6.29
95	2.89	2.85	2.22	0
100	2	2.1	2	0
Sum of squared deviations:		0.01	219.85	82.02

ANNEXURE-2

Interpolated standard life expectancies by single year obtained from Coal and Guo model life tables using piece wise exponential method. 9CGWM26: Coal and Guo Model West Female Level 25 used as standard for Males. CGWF26: Coal and Guo Model West Female Level 26 used as standard for Females.

Age	CGWM26	CGWF26	Age	CGWM26	CGWF26	Age	CGWM26	CGWF26
0	80	82.5	34	47.16	49.36	68	16.35	17.91
1	79.48	81.83	35	46.19	48.38	69	15.54	17.06
2	78.52	80.86	36	45.23	47.41	70	14.72	16.2
3	77.55	79.89	37	44.26	46.44	71	14.02	15.44
4	76.59	78.92	38	43.3	45.47	72	13.32	14.66
5	75.62	77.95	39	42.33	44.49	73	12.59	13.88
6	74.63	76.96	40	41.36	43.52	74	11.85	13.09
7	73.65	75.96	41	40.41	42.56	75	11.1	12.28
8	72.66	74.97	42	39.46	41.6	76	10.53	11.64
9	71.67	73.98	43	38.51	40.64	77	9.94	10.68
10	70.68	72.98	44	37.56	39.68	78	9.33	10.31
11	69.7	71.99	45	36.61	38.72	79	8.69	9.61
12	68.71	71	46	35.68	37.77	80	8.03	8.89
13	67.72	70	47	34.75	36.83	81	7.62	8.42
14	66.73	69.01	48	33.81	35.88	82	7.18	7.91
15	65.74	68.01	49	32.88	34.94	83	6.71	7.38
16	64.76	67.03	50	31.94	33.99	84	6.19	6.81
17	63.77	66.04	51	31.04	33.07	85	5.64	6.2
18	62.79	65.05	52	30.13	32.15	86	5.37	5.89
19	61.81	64.06	53	29.22	31.22	87	5.07	5.54
20	60.83	63.08	54	28.31	30.29	88	4.73	5.15
21	59.85	62.1	55	27.4	29.36	89	4.33	4.71
22	58.87	61.11	56	26.52	28.46	90	3.88	4.21
23	57.89	60.13	57	25.64	27.56	91	3.74	4.04
24	56.92	59.15	58	24.75	26.65	92	3.55	3.82
25	55.94	58.17	59	23.86	25.74	93	3.32	3.56
26	54.96	57.19	60	22.97	24.83	94	3.04	3.24
27	53.98	56.21	61	22.14	23.96	95	2.68	2.85
28	53.01	55.23	62	22.19	23.08	96	2.62	2.78
29	52.03	54.25	63	20.44	22.2	97	2.54	2.68
30	51.05	53.27	64	19.59	21.32	98	2.43	2.55
31	50.08	52.29	65	18.73	20.43	99	2.27	2.36
32	49.11	51.31	66	17.94	19.6	100	2.04	2.1
33	48.14	50.34	67	17.15	18.76			